Chapter 22

CORPORATE FAILURE: DEFINITIONS, METHODS, AND FAILURE PREDICTION MODELS

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Abstract

The exposure of a number of serious financial frauds in high-performing listed companies during the past couple of years has motivated investors to move their funds to more reputable accounting firms and investment institutions. Clearly, bankruptcy, or corporate failure or insolvency, resulting in huge losses has made investors wary of the lack of transparency and the increased risk of financial loss. This article provides definitions of terms related to bankruptcy and describes common models of bankruptcy prediction that may allay the fears of investors and reduce uncertainty. In particular, it will show that a firm filing for corporate insolvency does not necessarily mean a failure to pay off its financial obligations when they mature. An appropriate risk-monitoring system, based on well-developed failure prediction models, is crucial to several parties in the investment community to ensure a sound financial future for clients and firms alike.

Keywords: corporate failure; bankruptcy; distress; receivership; liquidation; failure prediction; Discriminant Analysis (DA); Conditional Probability Analysis (CPA); hazard models; misclassification cost models

22.1. Introduction

The financial stability of firms is of concern to many agents in society, including investors, bankers, governmental and regulatory bodies, and auditors. The credit rating of listed firms is an important indicator, both to the stock market for investors to adjust stock portfolios, and also to the capital market for lenders to calculate the costs of loan default and borrowing conditions for their clients. It is also the duty of government and the regulatory authorities to monitor the general financial status of firms in order to make proper economic and industrial policy. Further, auditors need to scrutinize the going-concern status of their clients to present an accurate statement of their financial standing. The failure of one firm can have an effect on a number of stakeholders, including shareholders, debtors, and employees. However, if a number of firms simultaneously face financial failure, this can have a wide-ranging effect on the national economy and possibly on that of other countries. A recent example is the financial crisis that began in Thailand in July 1997, which affected most of the other Asia-Pacific countries. For these reasons, the development of theoretical bankruptcy prediction models, which can protect the market from unnecessary losses, is essential. Using these, governments are able to develop policies in time to maintain industrial cohesion and minimize the damage caused to the economy as a whole.

Several terms can be used to describe firms that appear to be in a fragile financial state. From standard textbooks, such as Brealey et al. (2001) and Ross et al. (2002), definitions are given of distress, bankruptcy, or corporate failure. Pastena and Ruland (1986, p. 289) describe this condition as when

1. the market value of assets of the firm is less than its total liabilities;
2. the firm is unable to pay debts when they come due;
3. the firm continues trading under court protection.

Of these, insolvency, or the inability to pay debts when they are due, has been the main concern in the majority of the early bankruptcy literature. This is because insolvency can be explicitly identified and also serves as a legal and normative definition of the term "bankruptcy" in many developed countries. However, the first definition is more complicated and subjective in the light of the different accounting treatments of asset valuation. Firstly, these can give a range of market values to the company's assets and second, legislation providing protection for vulnerable firms varies between countries.

22.2. The Possible Causes of Bankruptcy

Insolvency problems can result from endogenous decisions taken within the company or a change in the economic environment, essentially exogenous factors. Some of the most common causes of insolvency are suggested by Rees (1990):

- Low and declining real profitability
- Inappropriate diversification: moving into unfamiliar industries or failing to move away from declining ones
- Import penetration into the firm's home markets
- Deteriorating financial structures
- Difficulties controlling new or geographically dispersed operations
- Over-trading in relation to the capital base
- Inadequate financial control over contracts
- Inadequate control over working capital
- Failure to eliminate actual or potential loss-making activities
- Adverse changes in contractual arrangements.

Apart from these, a new company is usually thought to be riskier than those with longer history. Blum (1974, p. 7) confirmed that "other things being equal, younger firms are more likely to fail than older firms." Hudson (1987), examining a sample between 1978 and 1981, also pointed out that companies liquidated through a procedure of creditors' voluntary liquidation or compulsory liquidation during that period were on average two to four years old and three-quarters of them less than ten years old. Moreover, Walker (1992, p. 9) also found that "many new companies fail within the first three years of their existence." This evidence suggests that the distribution of the failure likelihood against the company's age is positively skewed. However, a clear-cut point in age structure has so far not been identified to distinguish "new" from "young" firms in a business context, nor is there any convincing evidence with respect to the propensity to fail by firms of different ages. Consequently, the age characteristics of liquidated companies can only be treated as an observation rather than theory.

However, although the most common causes of bankruptcy can be noted, they are not sufficient to explain or predict corporate failure. A company with any one or more of these characteristics is not certain to fail in a given period of time. This is because factors such as government intervention may play an important role in the rescue of distressed firms. Therefore, as Bulow and Shoven (1978) noted, the conditions under which a firm goes through liquidation are rather complicated. Foster (1986, p. 535) described this as "there need not be a one-to-one correspondence between the non-distressed/distressed categories and the non-bankrupt/bankrupt categories." It is noticeable that this ambiguity is even more severe in the not-for-profit sector of the economy.

22.3. Methods of Bankruptcy

As corporate failure is not only an issue for company owners and creditors but also the wider economy, many countries legislate for formal bankruptcy procedures for the protection of the public interest, such as Chapter M and Chapter K in the US, and the Insolvency Act in the UK. The objective of legislation is to "[firstly] protect the rights of creditors ... [secondly] provide time for the distressed business to improve its situation ... [and finally] provide for the orderly liquidation of assets" (Pastena and Ruland, 1986, p. 289). In the UK, where a strong rescue culture prevails, the Insolvency Act contains six separate procedures, which can be applied to different circumstances to prevent either creditors, shareholders, or the firm as a whole from unnecessary loss, thereby reducing the degree of individual as well as social loss. They will be briefly described in the following section.

22.3.1. Company Voluntary Arrangements

A voluntary arrangement is usually submitted by the directors of the firm to an insolvency practitioner, "who is authorised by a recognised professional body or by the Secretary of State" (Rees, 1990, p. 394) when urgent liquidity problems have been identified. The company in distress then goes through the financial position in detail with the practitioner and discusses the practicability of a proposal for corporate restructuring. If the practitioner endorses the proposal, it will be put to the company's creditors in the creditors' meeting, requiring an approval rate of 75 percent of attendees. If the restructuring report is accepted, those notified will thus be bound by this agreement and the practitioner becomes the supervisor of the agreement. It is worth emphasizing that a voluntary arrangement need not pay all the creditors in full but a proportion of their lending (30 percent in a typical voluntary agreement in the UK) on a regular basis for the following several months. The advantages of this procedure are that it is normally much cheaper than formal liquidation proceedings and the creditors usually receive a better return.

22.3.2. Administration Order

It is usually the directors of the insolvent firm who petition the court for an administration order. The court will then assign an administrator, who will be in charge of the daily affairs of the firm. However, before an administrator is appointed, the company must convince the court that the making of an order is crucial to the survival of the company or for a better realization of the company's assets than would be the case if the firm were declared bankrupt. Once it is rationalized, the claims of all creditors are effectively frozen. The administrator will then submit recovery proposals to the creditors' meeting for approval within three months of the appointment being made. If this proposal is accepted, the administrator will then take the necessary steps to put it into practice.

An administration order can be seen as the UK version of the US Chapter K in terms of the provision of a temporary legal shelter for troubled companies. In this way, they can escape future failure without damaging their capacity to continue to trade (Counsell, 1989). This does sometimes lead to insolvency avoidance altogether (Homan, 1989).

22.3.3. Administrative Receivership

An administration receiver has very similar powers and functions as an administrator but is appointed by the debenture holder (the bank), secured by a floating or fixed charge after the directors of the insolvent company see no prospect of improving their ability to honor their debts. In some cases, before the appointment of an administration receiver, a group of investigating accountants will be empowered to examine the real state of the company. The investigation normally includes the estimation of the valuable assets and liabilities of the company. If this group finds that the company has no other choices but to be liquidated, an administration receiver will work in partnership with the investigation team and thus be entitled to take over the management of the company. The principal aim is to raise money to pay debenture holders and other preferential creditors by selling the assets of the businesses at the best price. The whole business may be sold as a going concern if it is worth more as an entity. As in an administration order, the receiver must advise creditors of any progress through a creditors' meeting, which is convened shortly after the initial appointment.

22.3.4. Creditors' Voluntary Liquidation

In a creditors' voluntary liquidation, the directors of the company will take the initiative to send an insolvency practitioner an instruction that will lead to the convening of a creditors' and shareholders' meetings. In a shareholders' meeting, a liquidator will be appointed and this is ratified in a subsequent creditors' meeting. Creditors have the right to determine who acts as liquidator. A liquidator will start to find potential purchasers and realise the assets of the insolvent firm in order to clear its debts. Unlike receivers who have wide ranging powers in the management of the businesses, the liquidator's ability to continue trading is restricted. This is the most common way to terminate a company (Rees, 1990).

22.3.5. Members' Voluntary Liquidation

The procedure for a member's voluntary liquidation is similar to that of the creditors' voluntary liquidation. The only difference is that in a members' voluntary liquidation the directors of the firm must swear a declaration of solvency to clear debts with fair interest within 12 months and creditors are not involved in the appointment of a liquidator. Therefore, a company's announcement of a members' voluntary liquidation by no means signals its insolvency, but only means closure with diminishing activity, purely a necessity to remain in existence.

22.3.6. Compulsory Liquidation

A compulsory liquidation is ordered by the court to wind up a company directly. This order is usually initiated by the directors of the insolvent firm or its major creditors. Other possible petitioners include the Customs and Excise, the Inland Revenue, and local government (Hudson, 1987, p. 213). The entire procedure is usually started with a statutory demand made by creditors who wish to initiate a compulsory liquidation. If the firm fails to satisfy their request in a stated period of time, this failure is sufficient grounds to petition the court to wind up the firm. Once the order is granted, the Official Receiver will take control of the company immediately or a liquidator will be appointed by the Official Receiver. The company then must cease trading and liquidation of assets begins. However, an interesting phenomenon is that many valuable assets may be removed or sold prior to the liquidator taking control, or even during the delivery of the petition to the court, leaving nothing valuable for the liquidator to deal with. In this sense, the company initiating a compulsory liquidation has been terminated in practical terms far before a court order is granted.

22.4. Prediction Model for Corporate Failure

Because corporate failure is not simply the closure of a company but has wider implications, it is important to construct models of corporate failure for assessment and prediction. If bankruptcy can be predicted accurately, it may be possible for the firm to be restructured, thus avoiding failure. This would benefit owners, employees, creditors, and shareholders alike.

There is an established literature that supports the prediction of corporate failure using financial ratio analysis. This is because by using financial performance data it is possible to control for the systematic effect of firm size and industry effects (Lev and Sunder, 1979, pp.187–188) in cross-section models to determine if there are signs of corporate failure. Thus, there is a history of financial ratio analysis in bankruptcy prediction research.

22.4.1. *Financial Ratio Analysis and Discriminant Analysis*

The earliest example of ratio analysis in predicting corporate failure is attributed to Patrick (1932), although it attracted more attention with the univariate studies of Beaver (1966). This work systematically categorized 30 popular ratios into six groups, and found that some ratios, such as cash flow/total debt ratio, demonstrated excellent predictive power in corporate failure models. These results also showed the deterioration of the distressed firms prior to failure, including a fall in net income, cash flow, and working capital, as well as an increase in total debt. Although this was a useful beginning, univariate analysis was later found to be limited and better results were obtained from including a number of ratios that combined to give a more robust model with improved predictive power.

With the increased popularity of the multi-ratio analysis, multivariate discriminant analysis (MDA) began to dominate the bankruptcy prediction literature from the 1980s. MDA determines the discriminant coefficient of each of the characteristics chosen in the model on the basis that these will discriminate efficiently between failed and nonfailed firms. A single score for each firm in the study is generated and a cut-off point determined that minimizes the dispersion of scores associated with firms in each category, including the probability of overlap between them. An intuitive advantage of MDA is that the model considers the entire profile of characteristics and their interaction. Another advantage lies in its convenience in application and interpretation (Altman, 1983, pp. 102–103).

One of the most popular MDA applications is the Z-score model developed by Altman (1968). Because of the success of the Z-score in predicting failure, 22 selected financial ratios were classified into five bankruptcy-related categories. In a sample of 33 bankrupt and 33 nonbankrupt manufacturing companies between 1946 and 1965, the final specification model determined the five variables, which are still frequently used in the banking and business sectors. The linear function is

$$Z\text{-score} = 1.2Z_1 + 1.4Z_2 + 3.3Z_3 + 0.6Z_4 + 0.999Z_5 \quad (22.1)$$

where

Z-score = overall index;
Z_1 = working capital/total assets;
Z_2 = retained earnings/total assets;
Z_3 = earnings before interest and taxes/total assets;
Z_4 = market value of equity/book value of total debt;
Z_5 = sales/total assets.

Altman (1968) also tested the cut-off point to balance Type I and Type II errors, and found that in general, it was possible for a company with a Z-score smaller than 1.8 to fail during the next few years whereas one with a Z-score higher than 2.99 was much more likely to succeed. The Z-score model remains popular as an indicator of credit risk for banks and other lenders.

Although these statistical discrimination techniques are popular in predicting bankruptcy, there are a number of methodological problems associated with them. Some are a function of the properties of financial ratios, for example, proportionality and zero-intercept assumptions are both critical to the credibility of the ratio analysis. The basic ratio form is assumed to be $y/x = c$, where y and x are two accounting variables that are different but linearly related and c is the value of the ratio. This raises three questions. First, is there an error term in the relationship between the two accounting variables? Second, is an intercept term likely to exist in this relationship? And finally, supposing the numerator and denominator are not linearly related?

With respect to the first question, Lev and Sunder (1979) proved that if there is an additive error term in the relationship between y and x suggested by the underlying theory, that is, $y = \beta x + e$ or $y/x = \beta + e/x$, the comparability of such ratios will be limited. This is because "the extent of deviation from perfect size control depends on the properties of the error term and its relation to the size variable, x" (Lev and Sunder, 1979, p. 191). The logic is as follows: Where the error term is homoscedastic, e/x is smaller for large firms than for small ones because x as a size variable for large firms will, on average, be greater than that of small firms. Therefore, the ratio y/x for large firms will be closer to the slope term β than that for small firms. Then, since the variance of the ratio y/x for smaller firms is greater than that of larger firms, it proves that the ratio y/x of two groups (large and small firms) are statistically drawn from two different distributions. This weakens the validity of the comparison between ratios. Furthermore, to include an additive error term in the relationship between the numerator and the denominator is not adequate as a size control.

However, if y is heteroscedastic, it may result in the homoscedasticity of y/x. But it is also possible that this heteroscedastic problem of y/x remains unchanged. Lev and Sunder (1979) note that this problem may be ameliorated only when the error term is multiplicative in the relationship, that is, $y = \beta xe$ or $y/x = \beta e$. This is because the deviation of y/x now has no mathematical relationship with the size variable x. As a result, this form of the ratio is more appropriate for purposes of comparison.

The same argument can be applied where an intercept term exists in the relationship between two ratio variables, represented by $y = \alpha + \beta x$ or $y/x = \beta + \alpha/x$. It is clear that the variance of y/x for smaller firms will be larger than that for larger firms under the influence of the term α/x. Again, this is not appropriate in comparisons of corporate performance.

If two variables are needed to control for the market size of y, such as $y = \alpha + \beta x + \delta z$, or $y = \alpha + \beta x + \delta x^2$ if the underlying relationship is nonlinear, the interpretation of the ratios can be ambiguous. All those problems cast doubt on the appropriateness of ratios in a number of situations. Theoretically, use of ratios is less problematic if and only if highly restrictive assumptions are satisfied. Empirically, Whittington (1980) claimed that violation of the proportionality assumption of the ratio form is the most common problem in research using financial data, especially in a time-series analysis at firm level. McDonald and Morris (1984, p. 96) found that the proportionality assumption is better satisfied when a group of firms in a simple homogeneous industry is analyzed, otherwise some amendment of the form of the ratios will be necessary. However, the replacement of the basic form of the ratio with a more sophisticated one is not a solution. On the contrary, on average, the basic form of the ratio performed quite satisfactorily in empirical studies. Keasey and Watson (1991, p. 90) also suggested that possible violations of the proportionality assumptions can be ignored, and since no further theoretical advances have been made on the topic, basic ratio analysis is still common in bankruptcy research.

In addition to the flaws in the design of financial ratios, there are other methodological problems associated with the use of MDA. Of these, non-normality, inequality of dispersion matrices across all groups, and nonrandom sampling are the most prevalent. The violation of the normality assumption has been extensively discussed in the literature since the 1970s (Kshirsagar, 1971; Deakin, 1976; Eisenbeis, 1977; Amemiya, 1981; Frecka and Hopwood, 1983; Zavgren, 1985; Karels and Prakash, 1987). Non-normality results in biased tests of significance and estimated error rates. Studies on univariate normality of financial ratios found that these distributions tended to be skewed (Deakin, 1976; Frecka and Hopwood, 1983; Karels and

Prakash, 1987). If the ratios included in the model are not perfectly univariate normal, their joint distribution will, a priori, not be multivariate normal (Karels and Prakash, 1987). Therefore, data used in bankruptcy modeling should seek to minimize multivariate non-normality problems. The traditional stepwise procedure does not satisfy this requirement. However, despite several complementary studies on data transformation and outlier removal for ratio normality (Eisenbeis, 1977; Ezzamel et al., 1987; Frecka and Hopwood, 1983), this is rarely used in MDA models (Shailer, 1989, p. 57). Because all these techniques are imperfect, McLeay (1986) advocated that selecting a better model is more straightforward than the removal of outliers or data transformations.

Given the problems of non-normality, inequality of dispersion matrices across all groups in MDA modeling is trivial by comparison. In theory, the violation of the equal dispersion assumption will affect the appropriate form of the discriminating function. After testing the relationship between the inequality of dispersions and the efficiency of the various forms of classification models, a quadratic classification rule seems to outperform a linear one in terms of the overall probability of misclassification when the variance–covariance matrices of the mutually exclusive populations are not identical (Eisenbeis and Avery, 1972; Marks and Dunn, 1974; Eisenbeis, 1977). More importantly, the larger the difference in dispersion across groups, the more the quadratic form of the discriminating function is recommended.

One of the strict MDA assumptions is random sampling. However, the sampling method used in bankruptcy prediction studies is choice-based, or state-based, sampling which results in an equal or approximately equal draw of observations from each population group. Because corporate failure is not a frequent occurrence (Altman et al., 1977; Wood and Piesse, 1988), such sampling technique will cause a relatively lower probability of misclassifying distressed firms as nondistressed (Type I Error) but a higher rate of misclassifying nondistressed firms as distressed (Type II Error) (Lin and Piesse, 2004; Kuo et al., 2002; Palepu, 1986; Zmijewski, 1984). Therefore, the high predictive power of MDA models claimed by many authors appears to be suspect. Zavgren (1985, p. 20) commented that MDA models are "difficult to assess because they play fast and loose with the assumptions of discriminant analysis." Where there is doubt about the validity of the results of MDA models, a more robust approach such as conditional probability analysis (CPA) is an alternative.

22.4.2. Conditional Probability Analysis

Since the late 1970s, the use of discriminant analysis has been gradually replaced by the CPA. This differs from MDA in that CPA produces the "probability of occurrence of a result, rather than producing a dichotomous analysis of fail/survive as is the norm with basic discriminant techniques" (Rees, 1990, p. 418). CPA primarily refers to logit and probit techniques and has been widely used in bankruptcy research (Keasey and Watson, 1987; Martin, 1977; Mensah, 1983; Ohlson, 1980; Peel and Peel, 1987; Storey et al., 1987; Zavgren, 1985, 1988). The major advantage of CPA is that it does not depend on the assumptions demanded by MDA (Kennedy, 1991, 1992). However, logit CPA is not always preferred under all conditions. If the multivariate normality assumption is met, the MDA Maximum Likelihood Estimator (LME) is more asymptotically efficient than MLE logit models. In all other circumstances, the MLE of MDA models may not be consistent, unlike that of logit models (Amemiya, 1981; Judge et al., 1985; Lo, 1986). However, as the rejection of normality in bankruptcy literature is very common, the logit model is appealing. Empirically, the logit analysis is most robust in the classification of distress.

The most commonly cited example of CPA research in this field is Ohlson (1980). The sample used included 105 bankrupt and 2058 nonbankrupt industrial companies during 1970–1976, contrasting with earlier studies that used equal numbers of bankrupts and nonbankrupts (Altman, 1968). The CPA logit analysis results in prediction failure with an accuracy rate of over 92 percent and included financial ratios to account for company size, capital structure, return on assets, and current liquidity, among others. This model was specified as:

$$Y = -1.3 - 0.4Y_1 + 6.0Y_2 - 1.4Y_3 + 0.1Y_4 - 2.4Y_5 - 1.8Y_6 + 0.3Y_7 - 1.7Y_8 - 0.5Y_9 \tag{22.2}$$

where:

Y = overall index;
Y_1 = log(total assets/GNP price-level index);
Y_2 = total liabilities/total assets;
Y_3 = working capital/ total assets;
Y_4 = current liabilities/current assets;
Y_5 = one if total liabilities exceed total assets, zero otherwise;
Y_6 = net income/total assets;
Y_7 = funds provided by operations/total liabilities;
Y_8 = one if net income was negative for the last two years, zero otherwise;
Y_9 = change in net income.

It is interesting to note that Ohlson (1980) chose 0.5 as the cut-off point, implicitly assuming a symmetric loss function across the two types of classification errors. The cut-off point was calculated using data beyond the estimation period, although the characteristics of the CPA model, and the large sample size, neutralized any problems (Ohlson, 1980, p. 126). It is important to note that while this was a valid approach for cross-section comparisons, it could not be transferred to comparisons across different time periods. With respect to predictive accuracy rates, Ohlson (1980) found that the overall results of the logit models were no obvious improvement on those from the MDA. Hamer (1983) tested the predictive power of MDA and logit CPA, and concluded that both performed comparably in the prediction of business failure for a given data set. However, given the predictive accuracy rates were overstated in previous MDA papers, mainly due to the use of choice-based sampling, this comparison may be biased and the inferences from them could favor CPA. Apart from this, other factors discussed in this literature question these comparisons, citing differences in the selection of predictors, the firm matching criteria, the lead time, the estimation and test time periods, and the research methodology. Unless these factors are specifically controlled, any claim about the comparative advantages between CPA and MDA in terms of the predictive ability will not be robust.

In conclusion, CPA provides all the benefits of other techniques, including ease of interpretation, but also has none of the strict assumptions demanded by MDA. Thus, CPA can be claimed to be the preferred approach to bankruptcy classification.

22.4.3. Three CPA Models: LP, PM, and LM

Three commonly cited CPA models are: the linear probability model (LP), the probit model (PM), and the logit model (LM). This technique estimates the probability of the occurrence of a result, with the general form of the CPA equation stated as

$$\begin{aligned} \Pr(y=1) &= F(x,\beta) \\ \Pr(y=0) &= 1 - F(x,\beta) \end{aligned} \tag{22.3}$$

In this specification, y is a dichotomous dummy variable which takes the value of 1 if the event occurs and 0 if it does not, and $Pr()$ represents the probability of this event. $F()$ is a function of a regressor vector x coupled with a vector β of parameters to govern the behavior of x on the probability. The problem arises as to what distribution best fits the above equation. Derived from three different distributions, LP, PM, and LM are then chosen to determine the best fit.

LP is a linear regression model, which is simple but has two main problems in application. The first is the heteroscedastic nature of the error term. Recall the form of an ordinary LP, $Y = X'\beta + \varepsilon$, where Y is the probability of an outcome and X is a column of independent variables, β is the parameter vector, and ε is the error term. When an event occurs, $Y = 1$, $\varepsilon = 1 - X'\beta$;

but when it does not occur, $Y = 0$, $\varepsilon = (-X'\beta)$. The second error term is not normally distributed, so Feasible General Least Squares Estimation Procedure (FGLS) should be used to correct heteroscedasticity (Greene, 1997, p. 87).

A more serious problem is that LP cannot constrain Y to lie between 0 and 1, as a probability should. Amemiya (1981, p. 1486) then suggested the condition that $Y = 1$ if $Y > 1$ and $Y = 0$ if $Y < 0$. But this can produce unrealistic and nonsensical results. Therefore, LP is rarely used and is discarded in the present study.

In the discussion of qualitative response models, there is a lively debate about the comparative benefits of logit and probit models. Although logit models are derived from a logistic density function and probit models from a normal density function, these two distributions are almost identical except that the logistic distribution has thicker tails and a higher central peak (Cramer, 1991, p. 15). This means the probability at each tail and in the middle of the logistic distribution curve will be larger than that of the normal distribution. However, one of the advantages of using logit is its computational simplicity, shown here in the relevant formulae:

$$\text{Probit Model: Prob } (Y = 1) = \int_{-\infty}^{\beta' x} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-t^2/2} \mathrm{d}t = \Phi(\beta' x) \quad (22.4)$$

$$\text{Logit Model: Prob } (Y = 1) = \frac{\exp(\beta' x)}{1 + \exp(\beta' x)} = \frac{1}{1 + \exp(-\beta' x)} \quad (22.5)$$

where function $\Phi()$ is the standard normal distribution. The mathematical convenience of logit models is one of the reasons for its popularity in practice (Greene, 1997, p. 874).

With respect to classification accuracy of CPA models, some comparisons of the results produced from these two models suggest that they are actually indistinguishable where the data are not heavily concentrated in the tails or the center (Amemiya, 1981; Cramer, 1991; Greene, 1997). This finding is consistent with the difference in the shape of the two distributions from which PM and LM are derived. It is also shown that the logit coefficients are approximately $\pi/\sqrt{3} \approx 1.8$ times as large as the probit coefficients, implying that the slopes of each variable are very similar. In other words, "the logit and probit model results are nearly identical" (Greene, 1997, p. 878).

The choice of sampling methods is also important in CPA. The common sampling method in the bankruptcy literature is to draw a sample with an approximately equal number of bankrupts and nonbankrupts, usually referred to as the state-based sampling technique, and is an alternative to random sampling. Although econometric estimation usually assumes random sampling, the use of state-based sampling has an intuitive appeal. As far as bankruptcy classification models are concerned, corporate failure is an event with rather low probability. Hence, a random sampling method may result in the inclusion of a very small percentage of bankrupts but a very high percentage of nonbankrupts. Such a sample will not result in efficient estimates in an econometric model (Palepu, 1986, p. 6). In contrast, state-based sampling is an "efficient sample design" (Cosslett, 1981, p. 56), which can effectively reduce the required sample size without influencing the provision of efficient estimators if an appropriate model and modification procedure are used. Thus, in bankruptcy prediction, the information content of a state-based sample for model estimation is preferred to that of random sampling. A state-based sample using CPA resulted in an understatement of Type I errors but an overstatement of Type II errors (Palepu, 1986; Lin and Piesse, 2004).

Manski and McFadden (1981) suggested several alternatives that can minimize the problems of state-based sampling. These include the weighted exogenous sampling maximum likelihood estimator (WESMLE) and the modified version by Cosslett (1981), the nonclassical maximum likelihood estimator (NMLE), and the conditional maximum likelihood estimator (CMLE). They compare and report these estimation procedures, which can be summarized as follows:

- All these estimators are computationally tractable, consistent, and asymptotically normal.
- The weighted estimator and conditional estimator avoid the introduction of nuisance parameters.
- The nonclassical maximum likelihood estimators are strictly more efficient than the others in large samples.
- In the presence of computational constraints, WESMLE and CMLE are the best; otherwise, NMLE is the most desirable.

Thus, by using any of these modifications, the advantages of using state-based sampling technique can be retained, while the disadvantages can be largely removed. The inference from this comparison is that the selection of modification method depends upon two factors: the sample size and the computational complexity. The modification cited in the bankruptcy literature is CMLE for three main reasons. Firstly, it has been extensively demonstrated in logit studies by Cosslett (1981) and Maddala (1983). Secondly, it was the model of choice in the acquisition prediction model by Palepu (1986), the merger/insolvency model by BarNiv and Hathorn (1997), and the bankruptcy classification models by Lin and Piesse (2004). Finally, because CMLE only introduces a change to the constant term that normally results from MLE estimation, while having no effects on the other parameters, this procedure is relatively simple. Without bias caused by the choice of sampling methods, modified CPA can correct all the methodological flaws of MDA.

22.5. The Selection of an Optimal Cut-Off Point

The final issue with respect to the accuracy rate of a bankruptcy classification model is the selection of an optimal cut-off point. Palepu (1986) noted that traditionally the cut-off point determined in most early papers was arbitrary, usually 0.5. This choice may be intuitive, but lacks theoretical justification. Joy and Tollefson (1975), Altman and Eisenbeis (1978), and Altman et al. (1977) calculated the optimal cut-off point in the ZETA model. Two elements in the calculation can be identified, the costs of Type I and Type II errors and the prior probability of failure and survival, both of which had been ignored in previous studies. However, Kuo et al. (2002) uses fuzzy theory methods to improve a credit decision model.

Although their efforts were important, unsolved problems remain. The first is the subjectivity in determining the costs of Type I and Type II errors. Altman et al. (1977, p. 46) claimed that bank loan decisions will be approximately 35 times more costly when Type I errors occurred than for Type II errors. This figure is specific to the study and is not readily transferred and therefore a more general rule is required. The second problem is the subjectivity of selecting a prior bankruptcy probability. Wood and Piesse (1988) criticized Altman et al. (1977) for choosing a 2 percent higher failure rate than the annual average failure rate of 0.5 percent, suggesting spurious results from Altman et al. and necessitating a correction that was taken up in later research. The final problem is that the optimal cut-off score produced may not be "optimal" when multinormality and equal dispersion matrices assumptions are violated, which is a common methodological problem in this data analysis (Altman et al. 1977, p. 43, footnote 17).

The optimal cut-off equation in Maddala (1983, p. 80) is less problematic. It begins by developing an overall misclassification cost model:

$$C = C_1 P_1 \int_{G_2} f_1(x)\mathrm{d}x + C_2 P_2 \int_{G_1} f_2(x)\,\mathrm{d}x \tag{22.6}$$

where

C = the total cost of misclassification;
C_1 = the cost of mis-classifying a failed firm as a non-failed one (Type I error);
C_2 = the cost of mis-classifying a non-failed firm as a failed one (Type II error);

P_1 = the proportion of the failed firms to the total population;
P_2 = the proportion of the non-failed firms to the total population;
G_1 = the failed firm group;
G_2 = the non-failed firm group;
x = a vector of characteristics $x = (x_1, x_2, \ldots, x_k)$;
$f_1(x)$ = the joint distribution of the characteristics x in the failed group;
$f_2(x)$ = the joint distribution of x in the non-failed group.
$P_1 + P_2 = 1$

However,

$$\text{Given} \quad \int_{G_2} f_1(x)\mathrm{d}x + \int_{G_1} f_1(x)\mathrm{d}x = 1 \tag{22.7}$$

Combining (22.6) and (22.7) gives

$$\begin{aligned} C &= C_1P_1(1 - \int_{G_1} f_1(x)\mathrm{d}x) + C_2P_2 \int_{G_1} f_2(x)\mathrm{d}x \\ &= C_1P_1 + \int_{G_1} [C_2P_2f_2(x) - C_1P_1f_1(x)]\mathrm{d}\,x \end{aligned} \tag{22.8}$$

then to minimize the total cost of misclassification, min C, it is necessary for

$$C_2P_2f_2(x) - C_1P_1f_1(x) \leq 0 \tag{22.9}$$

or

$$\frac{f_1(x)}{f_2(x)} \geq \frac{C_2P_2}{C_1P_1} \tag{22.10}$$

If it is assumed that the expected costs of Type I error and Type II error are equal, $C_2P_2 = C_1P_1$, the condition to minimize the total misclassification cost will be

$$\frac{f_1(x)}{f_2(x)} \geq 1 \tag{22.11}$$

This result is consistent with that proposed by Palepu (1986), assuming equal costs of Type I and II errors. Therefore, the optimal cut-off point is the probability value where the two conditional marginal densities, $f_1(x)$ and $f_2(x)$, are equal. In this equation, there is no need to use the prior failure rate to calculate the optimal cut-off point, the *ex post* failure rate (that is, the sample failure rate). Palepu (1986) illustrates this more clearly using Bayes' theorem.

Instead of using the costs of Type I and Type II errors, the expected costs of these errors are still unknown. Unfortunately, the subjectivity of deciding the relationship between the two types of expected costs still remains. There is no theoretical reason why they should be the same. However, compared to the previous arbitrary 50 percent cut-off point, this assumption is neutral and therefore preferred. Examples of applications using this method to determine the cut-off probability can be found in Palepu (1986) and Lin and Piesse (2004).

22.6. Recent Developments

While MDA and CPA are classified as static analyses, dynamic modeling is becoming more common in the bankruptcy literature. Shumway (2001) criticized static bankruptcy models for their examination of bankrupt companies 1 year prior to failure, while ignoring changes in the financial status of the firm year to year and proposed a simple dynamic hazard model to assess the probability failure on a continuous basis. Given the historical infrequency of corporate failure, the hazard model avoids the small sample problem because it requires all available time series of firm information. Because the hazard model takes the duration dependence, time-varying covariates, and data sufficiency problems into consideration, it is methodologically superior to both the MDA and CPA family of models. More empirical evidence is needed on its predictive power. Similar studies are in Whalen (1991) and Helwege (1996).

22.7. Conclusion

There are many reasons why a firm may fail and corporate insolvency *does not necessarily* include the inability to pay off financial obligations when they mature. For example, a solvent company can also be wound up through a member's voluntary liquidation procedure to maximize the shareholders' wealth when the realized value of its assets exceeds its present value in use. Bulow and Shoven (1978) modeled the potential conflicts among the various claimants to the assets and income flows of the company (for example, bondholders, bank lenders, and equity holders) and found that a liquidation decision should be made when "the coalition of claimants with negotiating power can gain from immediate liquidation" (Bulow and Shoven, 1978, p. 454). Their model also considered the existence of some asymmetric claims on the firm. This emphasizes the complex nature of bankruptcy decisions and justifies the adoption of members' voluntary liquidation procedure to determine a company's future (see Brealey and Myers, 2001, p. 622; Ross and Westerfield, 2002, p. 857).

The evolution and development of failure prediction models have produced increasingly superior methods, although an increase of their predictive power does not necessarily correlate with complexity. In addition, the costs of bankruptcy vary with different institutional arrangements and different countries (Brealey and Myers, 2001, pp. 439–443; Ross and Westerfield, 2002, p. 426). This implies that a single bankruptcy prediction model, with a fixed cut-off probability that can be used for all time periods and in all countries, does not exist. This paper has raised some of the problems with modeling corporate failure and reviewed some empirical research in the field.

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